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High Pt hadron-hadron correlations

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Abstract. We propose the formulation of a dihadron fragmentation function in terms of parton matrix elements. Under the collinear factorization approximation and facilitated by the cut-vertex technique, the two hadron inclusive cross section at leading order (LO) in e+ e- annihilation is shown to factorize into a short distance parton cross section and the long distance dihadron fragmentation function. We also derive the DGLAP evolution equation of this function at leading log. The evolution equation for the non-singlet and singlet quark fragmentation function and the gluon fragmentation function are solved numerically with the initial condition taken from event generators. Modifications to the dihadron fragmentation function from higher twist corrections in DIS off nuclei are computed. Results are presented for cases of physical interest.

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1. Introduction

One of the most promising signatures for the formation of a quark gluon plasma (QGP) in a heavy-ion collision has been that of jet quenching [1]. This phenomenon leads to the suppression of high p_T particles emanating from such collisions. Such jet quenching phenomena have been among the most striking experimental discoveries from the Relativistic Heavy Ion Collider (RHIC) at Brookhaven National Laboratory. Simultaneously, the modification of jets in cold nuclear matter has been measured via the deep-inelastic scattering (DIS) of leptons off nuclei at the HERMES detector at the Deutsche Elektronen-Synchrotron (DESY). Such experiments, besides providing an essential baseline for jet modifications in hot matter, are interesting tests of nuclear enhanced higher twist effects on parton propagation in nuclei.

In the investigation of jet suppression in heavy-ion collisions, correlations between two high p_T hadrons in azimuthal angle are used to study the change of jet structure [2]. While the back-to-back correlations are suppressed in central $Au + Au$ collisions, indicating parton energy loss, the same-side correlations remain approximately the same as in $p + p$ and $d + Au$ collisions. An almost identical situation has been observed in the modification of fragmentation functions in cold nuclear matter: while the single inclusive measurements indicate a large suppression hadrons with large momentum fractions (large z), the ratio of the double inclusive measurements to the single inclusive measurements show minimal variation with atomic number. Given the experimental kinematics, this is considered as an indication of parton hadronization outside the medium. However, since the same-side correlation corresponds to two-hadron distribution within a single jet, the observed phenomenon is highly nontrivial. To answer the question as to why a parton with a reduced energy would give the

same two-hadron distribution, one has to take a closer look at the single and double hadron fragmentation functions and their modification in a medium. In this article, we present results for the evolution of the dihadron fragmentation function in the process of e^+e^- annihilation, and its modification in a cold nuclear medium. Modifications in a hot QGP will be presented in a later effort.

For reactions at an energy scale much above Λ_{QCD}, one can factorize the cross section into a short-distance parton cross section which is computable order by order as a series in $\alpha_s(Q^2)$ and a long-distance phenomenological object (the fragmentation function) which contains the non-perturbative information of parton hadronization [3]. These functions are process independent. If measured at one energy scale, they may be predicted for all other energy scales via the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution equations [4]. In this article, we will extend this formalism to the double inclusive fragmentation function $D_q^{h_1,h_2}(z_1, z_2, Q^2)$ or the dihadron fragmentation function, where z_1, z_2 represent the forward momentum fractions of the two hadrons. In the absence of experimental data, the dihadron fragmentation functions will be sampled at one energy scale from event generators and the prediction of the derived DGLAP evolution will be verified against the measurement at another energy scale. The dihadron fragmentation function will then be subjected to medium modification in a DIS scenario. Comparisons with data from the HERMES experiment will be presented.

2. The double fragmentation function and its evolution

We begin our analysis with the following semi-inclusive process

$$e^+ + e^- \to \gamma^* \to h_1 + h_2 + X$$

of e^+e^- annihilation. We consider two-jet events where both the identified hadrons h_1 and h_2 emanate from the same jet. At leading order in the strong coupling, this occurs from the conversion of the virtual photon into a back-to-back quark and anti-quark pair which fragment into two jets of hadrons.

In the limit of very large Q^2 of the reaction, we may invoke the collinear approximation. Under this approximation, at leading twist, we can demonstrate the factorization of the two-hadron inclusive cross section into a hard total partonic cross section σ_0 and the double inclusive fragmentation function $D_q^{h_1h_2}(z_1, z_2)$ (see Ref. [5] for details),

$$\frac{d^2\sigma}{dz_1 dz_2} = \sum_q \sigma_0^{q\bar{q}} \left[D_q^{h_1h_2}(z_1, z_2) + D_{\bar{q}}^{h_1h_2}(z_1, z_2) \right]. \tag{1}$$

In the above equation, the leading order double inclusive fragmentation function of a quark is obtained as

$$D_q^{h_1,h_2}(z_1, z_2) = \int \frac{dq_\perp^2}{8(2\pi)^2} \frac{z^4}{4z_1z_2} \int \frac{d^4p}{(2\pi)^4} \mathbf{Tr}\left[\frac{\not{n}}{2\mathbf{n}\cdot\mathbf{p}_h} \int d^4x e^{i\mathbf{p}\cdot\mathbf{x}} \sum_{S-2} \right.$$
$$\left. \langle 0|\psi_q^\alpha(x)|p_1p_2S-2\rangle\langle p_1p_2S-2|\bar{\psi}_q^\beta(0)|0\rangle \right] \delta\left(z - \frac{p_h^+}{p^+} \right). \tag{2}$$

In the above equation $z = z_1 + z_2$, $\mathbf{p}$ represents the momentum of the fragmenting parton, $\mathbf{p}_h$ is the sum of the momenta of the two detected hadrons *i.e.* $\mathbf{p}_1 + \mathbf{p}_2$, $q_\perp$ is

the relative transverse momentum between the two detected hadrons and **n** is a light-like null vector. The sum over S indicates a sum over all possible final hadronic states. The above equation may be represented by the diagrams of the cut vertex notation [6] as that in the left panel of Fig. 1. Note that all transverse momentum $q_\perp$ up to a scale $\mu_\perp$ have been integrated over into the definition of the fragmentation function. Hadrons with transverse momenta $\geq \mu_\perp$ may not emanate from the fragmentation of a single parton.

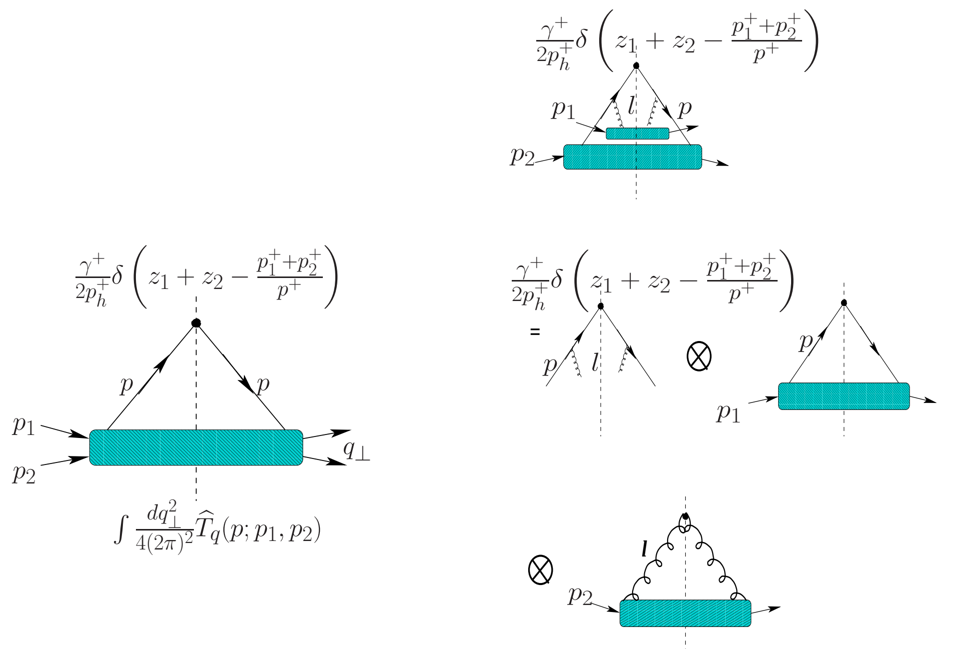

Figure 1. The left panel represents the cut-vertex representation of the dihadron fragmentation function. The right panel represents a Next-to-Leading order correction from quark and gluon single fragmentation.

In the interest of simplicity we first specialize to the case of the non-singlet fragmentation function, $D_{NS}^{h_1,h_2}(z_1,z_2) = D_q^{h_1,h_2}(z_1,z_2) - D_{\bar{q}}^{h_1,h_2}(z_1,z_2)$. With the definition of the dihadron fragmentation functions in the operator formalism, which is shown to factorize from the hard parton cross section at LO, we may now evaluate its DGLAP evolution by computing the double inclusive cross section at next to leading order (NLO). This constitutes a rather involved procedure; the reader is referred to Ref. [5] for details. The resulting DGLAP evolution consists of two parts. There remains the usual evolution due to the radiation of soft gluons and both detected hadrons emanating with a small transverse momentum off the same parton. Hadron pairs with transverse momentum larger than $\mu_\perp$ are produced perturbatively, by the independent single fragmentation of a quark and a gluon which emanate from the splitting of a quark. The contribution of this process to the evolution of the dihadron fragmentation function is represented by the cut-vertex diagram in the right panel of Fig. 1. Incorporating this diagram, we obtain the DGLAP evolution of the dihadron fragmentation function as

$$\frac{\partial D_{NS}^{h_1 h_2}(Q^2)}{\partial \log Q^2} = \frac{\alpha_s}{2\pi}\left[P_{q\to qg} * D_{NS}^{h_1 h_2}(Q^2) + \hat{P}_{q\to qg} \bar{*} D_{NS}^{h_1}(Q^2) D_g^{h_2}(Q^2) + 1 \to 2\right], \quad (3)$$

where the switch $1 \to 2$ is meant solely for the last term. The expressions for the splitting functions ($P_{q\to qg}$ and $\hat{P}_{q\to qg}$), single fragmentation functions, as well as the convolution notations ($*$ and $\bar{*}$) may be obtained from Ref. [5]. The above equation may be solved numerically and results for the cases of $z_1 = 2z_2$ and $3z_2$ are presented in Fig. 2. We assume a simple ansatz for the initial condition at $Q^2 = 2$ GeV^2 *i.e.* $D_{NS}^{h_1 h_2}(z_1, z_2) = D^{h_1}(z_1) D^{h_2}(z_2)$. We present results for the evolution with $\log(Q^2)$ at intervals of 1, up to $\log(Q^2) = 4.693$ *i.e.* $Q^2 = 109$ GeV^2. It may be noted that the results demonstrate minimal change as the energy scale is varied over a wide range of values.

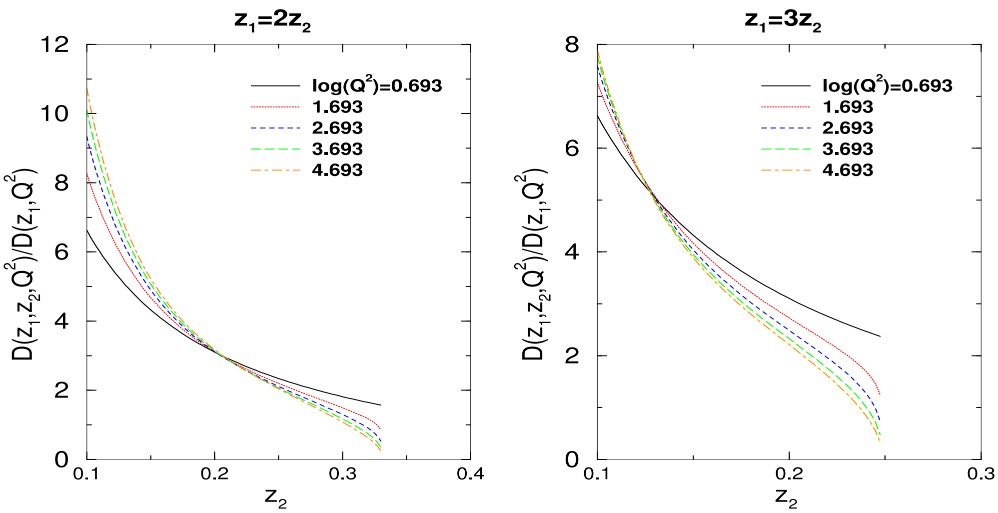

Figure 2. Results of the ratio of the non-singlet quark dihadron fragmentation function $D_q^{h_1 h_2}(z_1, z_2, Q^2)$ to the single leading fragmentation function $D_q^{h_1}(z_1, Q^2)$. In the left panel $z_1 = 2z_2$, in the right panel $z_1 = 3z_2$. Results are presented from $Q^2 = 2\text{GeV}^2$ to 109GeV^2.

In generalizing from the non-singlet quark fragmentation function to the quark or antiquark fragmentation function one simply includes a piece in which the fragmenting quark may radiate a gluon and both detected hadrons may emanate from the fragmentation of this gluon. However, this new piece couples the evolution of the quark fragmentation function with the evolution of the gluon fragmentation function. The evolution of the gluon fragmenation function has the usual components where the gluon may radiate further gluons prior to undergoing fragmentation, or convert into a quark-antiquark pair with the detected hadrons materializing from either parton. It also contains the new additional pieces where each of the detected hadrons may emanate from each of the radiated partons. We illustrate the various components of the gluon fragmentation function by the following equation (see Ref. [7] for details),

$$\frac{\partial D_g^{h_1 h_2}(Q^2)}{\partial \log Q^2} = \frac{\alpha_s}{2\pi}\left[\sum_{f=1}^{6} \hat{P}_{g\to f\bar{f}} * D_f^{h_1 h_2}(Q^2) + \sum_{f=1}^{3} P_{q\to f\bar{f}} \bar{*} D_f^{h_1}(Q^2) D_{\bar{f}}^{h_2}(Q^2) + 1 \to 2\right.$$

$$+ P_{g\to gg} * D_g^{h_1h_2}(Q^2) + P_{g\to gg} \bar{\bar{*}} D_g^{h_1}(Q^2) D_g^{h_2}(Q^2) \Bigg]. \quad (4)$$

In the above equation, f indicates the flavour of a fermion. In the first term, the sum runs from 1 to 6 indicating a sum over both quarks and antiquarks. In the second term the sum runs from 1 to 3 indicating only fermions. The case of quarks replaced with antiquarks is indicated with the switch 1 to 2 which only applies to this term. The remaining terms indicate splitting into two gluons followed by fragmentation function. Expressions for the splitting functions may be obtained from Ref. [7].

Once the fragmentation functions are measured at a given energy scale, their variation with energy scale can be predicted with the aid of the evolution equations mentioned above. To date, there remain no measurements of two particle correlations in e^+e^-collisions. However, Monte Carlo event generators such as JETSET have enjoyed great success as simulators of such collisions. The quark and gluon dihadron fragmentation at a scale $\mu^2 = 2\text{GeV}^2$ are extracted from two and three jet events in JETSET (see Ref. [7] for details). These are represented by the filled symbols (triangles for quarks and squares for gluons) in Fig. 3. In both cases, the momentum fraction of the leading hadron is set as $z_1 = 0.5$. Once measured, these fragmentation functions are then evolved with the aid of the evolution equations outlined above. Results of the evolution with $\log(Q^2)$ at intervals of 1, up to $\log(Q^2) = 4.693$ *i.e.* $Q^2 = 109$ GeV^2 are presented in the left panel of Fig. 3 as the various lines. The fragmentation functions are once again measured at the higher scale from JETSET and compared with the calculation (open triangles for quarks and open squares for the gluons). The excellent agreement between the evolution equation and the predictions from JETSET at the higher scale provide an important test of the derived DGLAP equation for the dihadron fragmentation function.

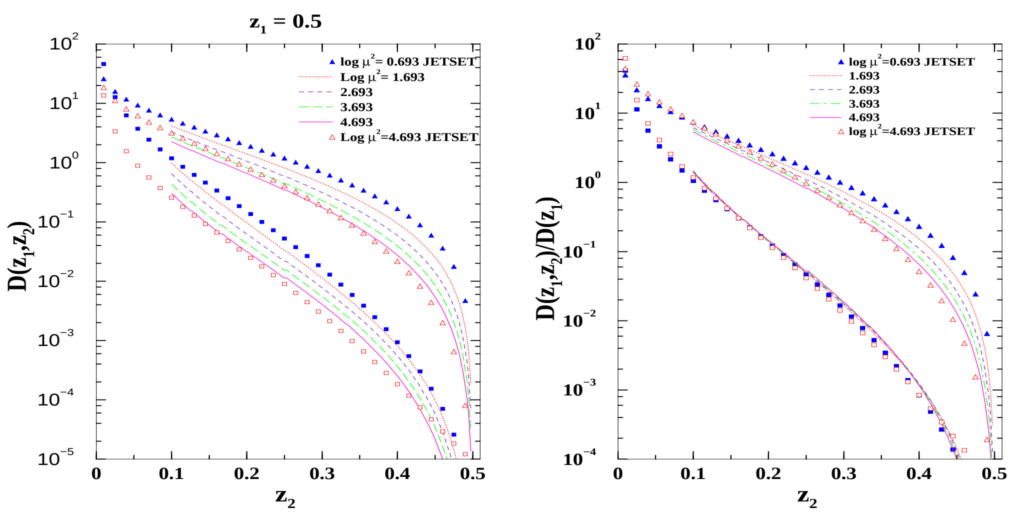

Figure 3. Left panel shows results of the evolution of the quark and gluon dihadron fragmentation functions. The right panel is a plot of the ratio of $D^{h_1h_2}(z_1, z_2, Q^2)$ to the single leading fragmentation function $D^{h_1}(z_1, Q^2)$, for the cases shown in the left panel. See text for details

The right panel of Fig. 3 shows the variation of the ratio $D(z_1, z_2, Q^2)/D(z_1, Q^2)$ with Q^2 while z_1 is held fixed at 0.5. While the quark dihadron fragmentation function

displays a slow variation, the gluon fragmentation function shows almost no change as the energy scale is varied over two orders of magnitude. This is consistent with the results obtained for the evolution of the non-singlet functions.

3. Medium Modification

The HERMES detector at DESY measures the semi-inclusive cross section for the production of one or two hadrons from electroproduction off nuclei such as Nitrogen and Krypton [8]. The measured quantity, $R_A^{h_1 h_2}$, is the number of events with at least two detected hadrons with the leading hadron assuming a momentum fraction of $z_1 \geq 0.5$, divided by the number of events with at least one hadron with a momentum fraction $z \geq 0.5$.

Due to various detector systematics, a double ratio is presented: $R_{A/D} = R_A^{h_1 h_2}/R_D^{h_1 h_2}$, where $R_D^{h_1 h_2}$ is the same ratio as discussed above but measured in scattering off Deuterium. Assuming that the Deuterium nucleus is two small to induce any noticeable medium effects on the fragmentation of hard partons passing through it and in the limit of low pair multiplicity per event, we equate the above ratio as

$$R_{A/D} = \frac{\tilde{D}_q^{h_1 h_2}(Q^2)/\tilde{D}_q^{h_1}(Q^2)}{D_q^{h_1 h_2}(Q^2)/D_q^{h_1}(Q^2)}, \tag{5}$$

where $D_q^{h_1 h_2}, D_q^{h_1}$ represent the quark fragmentation functions in vacuum, and $\tilde{D}_q^{h_1 h_2}, \tilde{D}_q^{h_1}$ represent those modified by the medium. Values of $R_{A/D}$ for Nitrogen are plotted in the right panel of Fig. 4.

The calculation of the medium modification of the fragmentation function is carried out in the nuclear enhanced higher twist formalism. This formalism is presented in detail in Ref. [9]. In principle, one calculates the cross section for the process where the recoiling lepton imparts a large momentum transfer (Q^2) to one of the quarks in the incoming nucleus via single photon exchange. The struck quark then undergoes multiple scattering off the soft gluon field of the nucleus prior to exiting the medium and fragmenting to hadrons. In this process it absorbs a certain amount of transverse momentum and virtuality, which it loses by gluon bremsstrahlung. Alternatively, the struck quark may be produced off shell and may radiate gluons prior to scattering off the soft gluons. In either case the radiated gluon (which also carries a colour charge) may also encounter further scattering of the gluon field prior to exiting the medium and fragmenting into a jet of hadrons. Interferences between the various processes mentioned above also need to be considered.

The eventual calculation of the medium modification involves the computation of multiple higher twist diagrams such as the one displayed in the left panel of Fig. 4. While these diagrams, which involve quark gluon correlations in nuclei, are suppressed by powers of Q^2, they receive an enhancement $\sim A^{\frac{1}{3}}$ from the fact that the quark and gluon may originate from any of the A nucleons in the nucleus (for further details see Refs. [9, 10]). Calculations for a Nitrogen nucleus are presented and compared to the HERMES data in the right panel of Fig. 4.

4. Discussions and Conclusions

This study was motivated by the observation [2] that the same side correlations of two high p_T hadrons in central $Au + Au$ collisions remain approximately unchanged

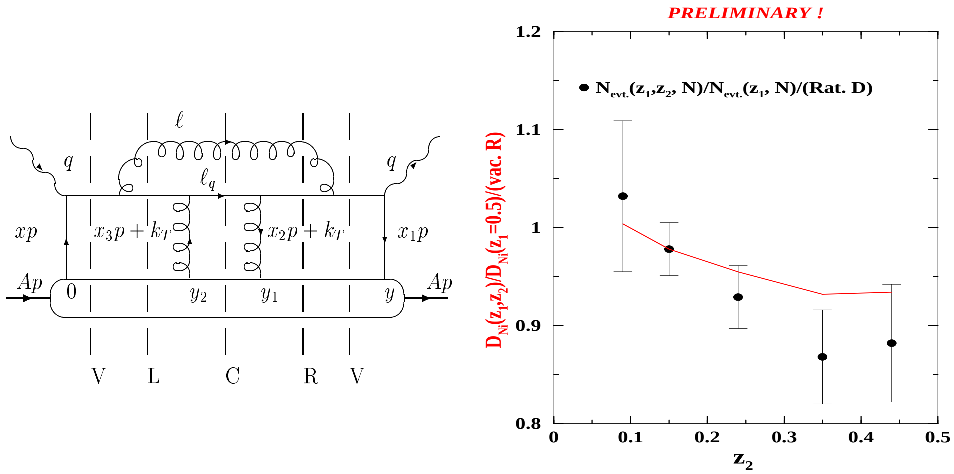

Figure 4. Left panel shows one of the higher twist diagrams involved in the calculation of the medium modification of the fragmentation functions. The various dashed lines indicate the various cuts: left (L), central (C), right (R) and virtual (V). The right panel shows the experimental data from HERMES for $R_{A/D}$ and the solid line is the calculation. Both the data and the calculation are preliminary.

as compared with that in $p+p$ collisions. A similar situation was also observed in DIS experiments. Neglecting the differences in production cross section and fragmentation functions for different parton species, the integrated yield of the same side correlation over a small range of angles should be the ratio of the dihadron to the single hadron fragmentation functions, $D_a^{h_1h_2}(z_1, z_2, Q^2)/D_a^{h_1}(z_1, Q^2)$. Thus, we evaluated the influence of DGLAP evolution on the ratio $D_q^{h_1h_2}(z_1, z_2, Q^2)/D_q^{h_1}(z_1, Q^2)$. Our numerical results indeed show little change of the ratio as Q^2 is varied over a wide range of values for both singlet and non-singlet quarks and for gluons.

We evaluated the influence of medium modification due to multiple scattering and induced gluon radiation on the above ratio in a cold nuclear medium. The results were found to be consistent, both with the experiment and also the vacuum evolution of the ratio. Calculations for the modification in a deconfined medium are in progress.

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